High order correlation functions for self interacting scalar field in de Sitter space

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We present the expressions of the three- and four-point correlation functions of a self interacting light scalar field in a de Sitter spacetime at tree order respectively for a cubic and a quartic potential. Exact expressions are derived and their limiting behaviour on super-horizon scales are presented. Their essential features are shown to be similar to those obtained in a classical approach.

I. INTRODUCTION

The computation of high order correlation functions of scalar fields in a de Sitter spacetime can be of interest for investigations of the physics of the early universe. There are indeed a growing number of indications that in its early phase the universe underwent an inflationary period [2] that can be accurately described by a de Sitter phase during which the energy density of the universe is thought to be dominated by the self energy of a scalar field [1]. Its fluctuations, or the fluctuations of any other light scalar fields, are thought to be the progenitors of the large-scale structure of the universe [3]. The statistical properties of the induced metric fluctuations then depend on the potential landscape in which the fields evolve. It is unlikely that the fluctuations produced along the inflaton direction can be significantly non-Gaussian as stressed in recent works [4, 5]. It is possible however that if there exist other self interacting light scalar fields during that period primordial non Gaussian metric fluctuations can be generated [6, 7, 8, 9]. The details of the effects induced by such self-interacting fields is based on the statistical properties of the metric fluctuations that quantum generated field fluctuations can induced.

The purpose of this paper is to investigate the foundation of these calculations by calculating the correlation properties of a test scalar field in a de Sitter background.

The content of the paper is the following. In the second part we present the basis of such computations. In the third we explore the expression of the leading order term of high order correlators for different self-interacting potentials. In particular we give explicit results for the superhorizon limit that corresponds to modes that can be observed today. Finally we give insights on the physical interpretations of those results.

II. CORRELATION FUNCTION COMPUTATIONS

A. Free field behavior

For the purpose of our calculations we assume that the inflationary phase can be described by a de Sitter background epoch. This description is only approximate since during the inflationary period the Hubble constant slowly varies with time but it is a framework in which all calculations can be pursued analytically. We have checked that to a large extent the results would not be affected if the background is changed [14].

A de Sitter spacetime [10] in flat spatial section slicing is described by the metric,

$$\mathrm{d}s^2 = \frac{1}{(H\eta)^2}\left(-\mathrm{d}\eta^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right) \tag{1}$$

which is conformal to half of the Minkowski spacetime. The conformal time is related to the cosmological time by

$$\eta = -\frac{1}{H}\mathrm{e}^{-Ht} \tag{2}$$

and runs from $-\infty$ to 0, the limit $\eta \to 0^-$ representing the "infinite future".

For a minimally coupled free quantum field of mass m, the solution can be decomposed in plane waves as

$$\widehat{v}_0(\mathbf{x},\eta) = \int \mathrm{d}^3\mathbf{k} \left[v_0(k,\eta)\widehat{b}_{\mathbf{k}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} + v_0^*(k,\eta)\widehat{b}_{\mathbf{k}}^\dagger \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}} \right] \tag{3}$$

where we have introduced $\widehat{v} \equiv a\,\widehat{\chi}$, a hat referring to a quantum operator. In this Heisenberg picture, the field has become a time-dependent operator expanded in terms of time-independent creation and annihilation operators satisfying the usual commutation relations $[\widehat{b}_{\mathbf{k}}, \widehat{b}_{\mathbf{k}'}^\dagger] = \delta_{\mathrm{Dirac}}(\mathbf{k}-\mathbf{k}')$. We can then define the free vacuum state by the requirement

$$\widehat{b}_{\mathbf{k}}\,|0\rangle = 0 \qquad \text{for all } \mathbf{k}. \tag{4}$$

As it is standard while working in curved spacetime [11], the definition of the vacuum state suffers from some arbitrariness since it depends on the choice of the set of modes $\chi_0(k,\eta)$. They satisfy the evolution equation

$$v_0'' + \left(k^2 - \frac{2}{\eta^2} - \frac{m^2/H^2}{\eta^2}\right) v_0 = 0, \tag{5}$$

the general solution of which is given by $\sqrt{\pi\eta/4}\left[c_1 H_\nu^{(1)}(k\eta) + c_2 H_\nu^{(2)}(k\eta)\right]$ with $|c_2|^2 - |c_1|^2 = 1$, where $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are the Hankel functions of first and second kind and with $\nu^2 = 9/4 - m^2/H^2$. Among this family of solutions, it is natural to choose the one enjoying the de Sitter symmetry and the same short distance behavior than in Minkowski spacetime. This leads to

$$v_0(k,\eta) = \frac{1}{2}\sqrt{\pi\eta}H_\nu^{(2)}(k\eta). \tag{6}$$

This uniquely defines a de Sitter invariant vacuum state referred to as the Bunch-Davies state vacuum [11]. In the massless limit, the solution (5) reduces to

$$v_0(k,\eta) = \left(1 - \frac{\mathrm{i}}{k\eta}\right)\frac{\mathrm{e}^{-\mathrm{i}k\eta}}{\sqrt{2k}}. \tag{7}$$

This result gives the expression of the equal-time two-point correlator of the Fourier modes,

$$\langle \chi(\mathbf{k}_1)\chi(\mathbf{k}_2)\rangle = \delta_{\mathrm{Dirac}}(\mathbf{k}_1+\mathbf{k}_2)\,P_2(k_1) \tag{8}$$

$$P_2(k_1) = \frac{H^2\eta^2}{2k_1}\left(1 + \frac{1}{k_1^2\eta^2}\right). \tag{9}$$

A remarkable result is that in the superhorizon limit ($k\eta \to 0$) is that the phase of $v_0(k,\eta)$ freezes and it reads,

$$v_0(k,\eta) = \frac{\mathrm{i}}{\eta}\frac{1}{\sqrt{2k^3}}. \tag{10}$$

Note that the Fourier modes of χ are frozen only in the case of a massless field in a de Sitter background. As a result the field χ behaves like a classic stochastic field with fluctuations whose 2-point correlator is,

$$\langle \chi(\mathbf{k}_1)\chi(\mathbf{k}_2)\rangle = \frac{H^2}{2\,k_1^3}\,\delta_{\mathrm{Dirac}}(\mathbf{k}_1+\mathbf{k}_2) \tag{11}$$

And since χ is a free field, its superhorizon fluctuations follow a Gaussian statistics.

B. Computation of higher-order correlation functions

Having determined the free field solutions, one can then express perturbatively the N-point correlation functions of the interacting field, χ, in terms of those of the free scalar field. The equal time correlators we are interested in are expectation values of product of field operators for the current time vacuum state. Such computations can be performed following general principles of quantum field calculations [4]. The simplest formulation it is to apply the

evolution operator $U(\eta_0,\eta)$ backward in time to transform the interacting field vacuum into the free field vacuum at an arbitrarily early time η_0 so that,

$$\langle v_{\mathbf{k}_1}\dots v_{\mathbf{k}_n}\rangle \equiv \langle 0|U^{-1}(\eta_0,\eta)\ v_{\mathbf{k}_1}\dots v_{\mathbf{k}_n}\ U(\eta_0,\eta)|0\rangle \tag{12}$$

where $|0\rangle$ is here the *free field* vacuum [15]. It is implicitly assumed in this expression that the coupling of the field χ is switched on at time η_0. We will see in the following that the choice of η_0 is not important as long as it is much earlier than any other times intervening in the problem.

The evolution operator U can be written in terms of the interaction Hamiltonian, H_I, as

$$U(\eta_0,\eta) = \exp\left(-\mathrm{i}\int_{\eta_0}^{\eta}\mathrm{d}\eta'\ H_I(\eta')\right) \tag{13}$$

If one is interested only in a single vertex interaction quantity, the evolution operator can be expanded to linear order in H_I,

$$U(\eta_0,\eta) = I_{\mathrm{d}} - \mathrm{i}\int_{\eta_0}^{\eta}\mathrm{d}\eta'\ H_I(\eta') \tag{14}$$

so that the connected part of the above ensemble average at a time η finally reads,

$$\langle v_{\mathbf{k}_1}\dots v_{\mathbf{k}_n}\rangle_c = -\mathrm{i}\int_{\eta_0}^{\eta}\mathrm{d}\eta'\ \langle 0|\left[v_{\mathbf{k}_1}\dots v_{\mathbf{k}_n}, H_I(\eta')\right]|0\rangle \tag{15}$$

where the brackets stand for the commutator.

The result is expressed in terms of the Green function

$$G(k,\eta,\eta') = \frac{1}{2k}\left(1-\frac{\mathrm{i}}{k\eta}\right)\left(1+\frac{\mathrm{i}}{k\eta'}\right)\exp[\mathrm{i}k(\eta'-\eta)] \tag{16}$$

defined as $\langle 0|v(\mathbf{k},\eta)v(\mathbf{k}',\eta')|0\rangle = \delta_{\mathrm{Dirac}}(\mathbf{k}+\mathbf{k}')G(k,\eta,\eta')$.

Before we proceed to explore explicit cases, let us note that as long as calculations are restricted to tree order, the very same calculation can be done assuming that χ is a classic stochastic field whose stochastic properties are initially those of the quantum free field. Not surprisingly one finds the same formal expressions!

III. EXACT RESULTS

Results can be given in a closed form for simple self-interacting potentials. We give in this section explicit results for cubic and quartic potentials. Because of its renormalization properties, quartic potential is a natural choice to consider. We will however see that the case of a cubic potential can be relevant when finite volume effects are taken into account (see Ref. [13] for more details on finite volume effects).

In the following we thus assume that H_I is of the form

$$H_I(\chi) = \int \mathrm{d}^3\mathbf{x}\sqrt{-g}\,\frac{\lambda}{p!}\chi^p, \tag{17}$$

and we are then interested in the computation of the leading order part of the connected part of the ensemble average of products of p Fourier modes of the fields.

In case of a quartic coupling, we have,

$$\langle v_{\mathbf{k}_1}\dots v_{\mathbf{k}_4}\rangle_c = -\mathrm{i}\lambda\delta_{\mathrm{Dirac}}(\mathbf{k}_1+\dots+\mathbf{k}_4)\int^{\eta}\mathrm{d}\eta'\left[G(k_1,\eta,\eta')\dots G(k_4,\eta,\eta') - G^*(k_1,\eta,\eta')\dots G^*(k_4,\eta,\eta')\right] \tag{18}$$

This expression is unfortunately quite cumbersome to compute. It depends, for symmetry reasons, on the norms of the four wave vectors k_1,...,k_4 in the following combinations

$$\pi_1 = \sum_i k_i \tag{19}$$

$$\pi_2 = \sum_{i<j} k_i\,k_j \tag{20}$$

$$\pi_3 = \sum_{i<j<k} k_i\,k_j\,k_k \tag{21}$$

$$\pi_4 = \sum_{i<j<k<l} k_i\,k_j\,k_k\,k_l \tag{22}$$

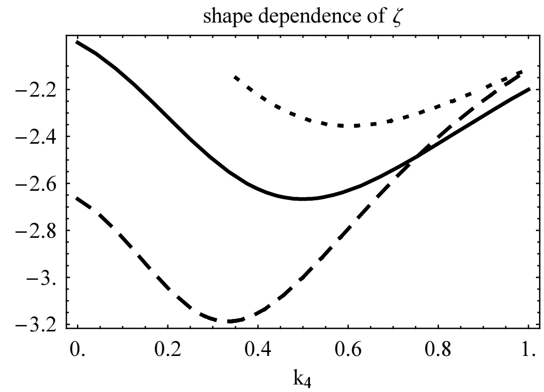

FIG. 1: Dependence of the function ζ as a function of k_4 for fixed values of k_1, k_2 and k_3. The solid line corresponds to configuration $k_1{=}0$, $k_2{=}\frac{1}{2}$, $k_3{=}\frac{1}{2}$, k_4; the dashed line to $k_1{=}\frac{1}{3}$, $k_2{=}\frac{1}{3}$, $k_3{=}\frac{1}{3}$, k_4 and the dotted line to $k_1{=}\frac{1}{6}$, $k_2{=}\frac{1}{3}$, $k_3{=}\frac{1}{6}$, k_4.

The expression (18) can then be rewritten as

$$\begin{aligned}
\langle v_{\mathbf{k}_1}\dots v_{\mathbf{k}_4}\rangle_c \;=\; & \frac{-\lambda}{24\,\pi_4^3\,\pi_1\,\eta^4}\,\delta_{\rm Dirac}\left(\sum_i \mathbf{k}_i\right)\Big(\\
& {\pi_1}^4 - 2\,{\pi_1}^2\,\pi_2 - \pi_1\,\pi_3 + 3\,\pi_4 + \left(-{\pi_1}^3\,\pi_3 + 3\,\pi_1\,\pi_2\,\pi_3 - {\pi_1}^2\,\pi_4 - 3\,\pi_2\,\pi_4\right)\eta^2 + 3\,{\pi_4}^2\,\eta^4 \\
& -\pi_1\,\left({\pi_1}^3 - 3\,\pi_1\,\pi_2 + 3\,\pi_3\right)\,\Big\{\left[\left(1-\pi_2\,\eta^2+\pi_4\,\eta^4\right)\,\cos(\pi_1\,\eta) + \eta\,\left(\pi_1 - \pi_3\,\eta^2\right)\,\sin(\pi_1\,\eta)\right]\,{\rm ci}(-\pi_1\,\eta) \\
& -\left[\eta\,\left(\pi_1-\pi_3\,\eta^2\right)\,\cos(\pi_1\,\eta) + \left(-1+\pi_2\,\eta^2-\pi_4\,\eta^4\right)\,\sin(\pi_1\,\eta)\right]\,{\rm si}(-\pi_1\,\eta)\Big\}\Big)
\end{aligned} \tag{23}$$

where the CosIntegral (ci) and SinIntegral (si) functions are defined, for $\eta < 0$, by

$$\int_{-\infty}^{\eta}\frac{{\rm d}\eta'}{\eta'}\,\exp[-{\rm i}k\eta'] \equiv {\rm ci}(-k\eta) + i\,{\rm si}(-k\eta). \tag{24}$$

The superhorizon limit (i.e. $k_i\eta \ll 1$ for all $i = 1\dots 4$) of Eq. (23) is

$$\langle v_{\mathbf{k}_1}\dots v_{\mathbf{k}_4}\rangle_c \;=\; \frac{-\lambda\;\delta_{\rm Dirac}\left(\sum_i \mathbf{k}_i\right)}{24\,\pi_4^3\,\pi_1\,\eta^4}\Big\{{\pi_1}^4 - 2\,{\pi_1}^2\,\pi_2 - \pi_1\,\pi_3 + 3\,\pi_4 - \pi_1\,\left({\pi_1}^3 - 3\,\pi_1\,\pi_2 + 3\,\pi_3\right)\,\left[\gamma + \log(-\pi_1\,\eta)\right]\Big\} \tag{25}$$

where γ is the Euler's constant ($\gamma \approx 0.577$). This implies that the 4-point correlator of the actual fields χ reads

$$\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_4}\rangle_c = -\,\frac{\lambda\,H^4}{24}\,\frac{\delta_{\rm Dirac}(\sum \mathbf{k}_i)}{\prod k_i^3}\;\left[-\sum k_i^3\left(\gamma + \zeta(\{k_i\}) + \log\left[-\eta\sum k_i\right]\right)\right] \tag{26}$$

In this expression, terms of the order of $k_i\eta$ have been neglected. This result illustrates the transition to the stochastic limit. The function ζ is an homogeneous function of the Fourier wave-numbers

$$\zeta(\{k_i\}) = \frac{-{\pi_1}^4 + 2\,{\pi_1}^2\,\pi_2 + \pi_1\,\pi_3 - 3\,\pi_4}{\pi_1\,\left({\pi_1}^3 - 3\,\pi_1\,\pi_2 + 3\,\pi_3\right)}. \tag{27}$$

The dependence of the function ζ on the wavelength ratios is illustrated on Fig. 1. It is found to be relatively weak. We did not fully explore its properties but one can explicitly show for instance that if one of the wave vectors vanishes, ζ is smaller than -2 and reaches its minimum ,$-8/3$, for a symmetric configuration of the three remaining wave vectors. From the plot on Fig. 1 it appears that the minimum value of ζ is -3.2 and is reached for a "square" configuration, that is when the four wavelengths are of equal length. The maximum value, -2, is reached when two of the wavelengths vanish. The overall variations of ζ with the wavelength ratios are therefore rather mild. It is then legitimate to describe the four-point correlation function as the sum of products of two-point power spectra, e.g.,

$$\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_4}\rangle_c \;=\; \delta_{\rm Dirac}\left(\sum \mathbf{k}_i\right)\,P_4(\{k_i\}) \tag{28}$$

$$P_4(\{k_i\}) \;=\; \nu_3(\{k_i\})\sum_i\prod_{j\neq i}\frac{H^2}{2k_j^3}. \tag{29}$$

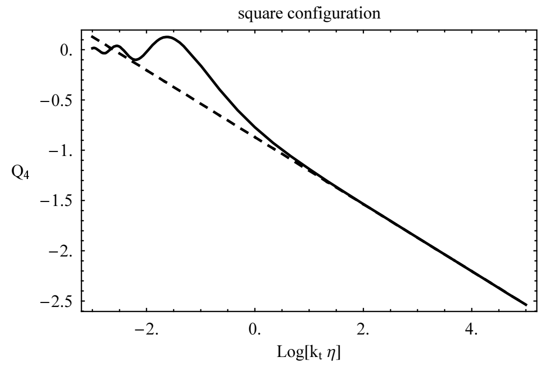

FIG. 2: Behaviour of the function Q_4 as of function of time. The transition to the superhorizon behavior (dashed line) is shown. The function Q_4 is shown here for a "square" configuration ($k_1 = k_2 = k_3 = k_4$) as a function of $k_t\eta = \sum k_i\eta$.

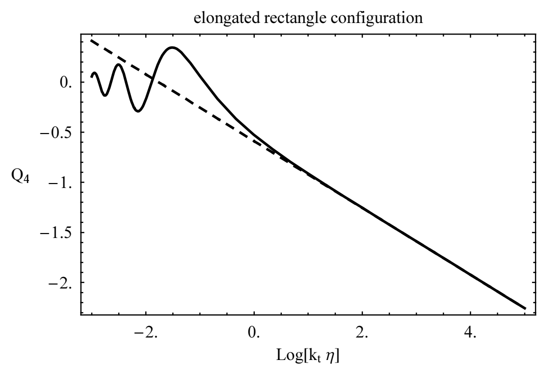

FIG. 3: Same as Fig. 2 for a "rectangular" configuration ($k_1 = k_2 = 4$, $k_3 = 4k_4$).

The vertex value, ν_3 is given by

$$\nu_3(\{k_i\}) = \frac{\lambda}{3H^2}\left[\gamma + \zeta(k_i) + \log\left(-\eta\sum k_i\right)\right] \tag{30}$$

that carries a weak geometrical dependence with the wave vectors geometry through ζ.

When the term $\log\left(-\eta\sum k_i\right)$ is large (and negative), that is when the number of e-folds, N_e, between the time of horizon crossing for the modes we are interested in and the end of inflation is large, the vertex value is simply given

$$\nu_3(\{k_i\}) = -\lambda\, N_e/(3\, H^2)$$

which corresponds exactly to the value that was obtained in Ref. [8] from the classical evolution of the stochastic field on superhorizon scales (see comments in last section).

The result obtained above is however more complete since it contains next to leading order terms. It allows, for instance, to estimate the validity regime of the superhorizon result. It shows in particular that the mode coupling induced on subhorizon scales are negligible as soon as the number of e-folds after horizon crossing exceeds a few units. This is made clear when one considers the reduced four-point correlation function, Q_4, defined as,

$$Q_4(\{k_i\}) = \frac{P_4(k_1,k_2,k_3,k_4)}{P_2(k_1)P_2(k_2)P_2(k_3) + \text{sym.}} \tag{31}$$

where P_2 is defined in Eq. (9). The behaviour of Q_4 as a function of time and for different configurations of the wavevectors is depicted on Figs. (2-4) and one can convince hismself that it converges rapidly toward the superhorizon result as soon as $-\log\sum k_i\eta \sim \mathcal{O}(1)$.

Similar results can be obtained for the 3-point function in the case of a cubic potential, $V(\chi) = \lambda\,\chi^3/3!$. In this case the quantity to compute is

$$\langle v_{\mathbf{k}_1}\dots v_{\mathbf{k}_3}\rangle = -\mathrm{i}\lambda\delta_{\mathrm{Dirac}}(\mathbf{k}_1+\ldots+\mathbf{k}_3)\int^{\eta}\frac{-\mathrm{d}\eta'}{H\eta'}\left[G(k_1,\eta,\eta')\ldots G(k_3,\eta,\eta') - G^*(k_1,\eta,\eta')\ldots G^*(k_3,\eta,\eta')\right]. \tag{32}$$

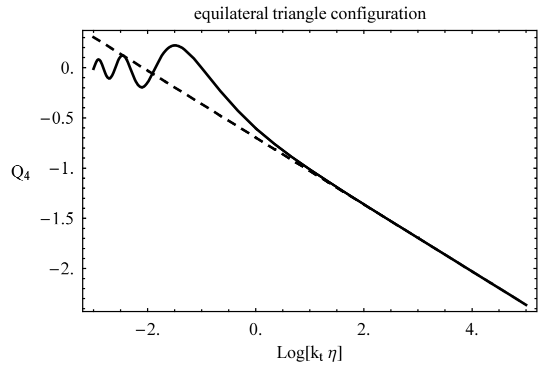

FIG. 4: Same as Fig. 2 for a "triangular" configuration ($k1 = 0, k_2 = k_3 = k_4$).

which gives,

$$\begin{aligned}\langle v_{\mathbf{k}_1}\dots v_{\mathbf{k}_3}\rangle &= \frac{\lambda}{12\,\pi_3^3\,\eta^3\,H}\,\delta_{\rm Dirac}(\mathbf{k}_1+\ldots+\mathbf{k}_3)\Big(\\ &\quad {\pi_1}^3-2\,\pi_1\,\pi_2-\pi_3+\left(-{\pi_1}^2\,\pi_3+3\,\pi_2\,\pi_3\right)\eta^2\\ &\quad -\left({\pi_1}^3-3\,\pi_1\,\pi_2+3\,\pi_3\right)\Big\{\,\left[\left(1-\pi_2\,\eta^2\right)\,\cos(\pi_1\,\eta)+\eta\,\left(\pi_1-\pi_3\,\eta^2\right)\,\sin(\pi_1\,\eta)\right]\,{\rm ci}(-\pi_1\,\eta)\\ &\quad -\left[\eta\,\left(\pi_1-\pi_3\,\eta^2\right)\,\cos(\pi_1\,\eta)+\left(-1+\pi_2\,\eta^2\right)\,\sin(\pi_1\,\eta)\right]\,{\rm si}(-\pi_1\,\eta)\Big\}\Big)\end{aligned} \tag{33}$$

To a factor 2η, one recovers basically the same expression as for the fourth cumulant (23) when π_4 is set to zero. In the superhorizon limit, the expression of this cumulant reads

$$\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_3}\rangle = -\,\frac{\lambda\,H^2}{12}\frac{\delta_{\rm Dirac}\left(\sum\mathbf{k}_i\right)}{\prod k_i^3}\,\left[-\sum k_i^3\left(\gamma+\zeta_3(\{k_i\})+\log\left[-\eta\sum k_i\right]\right)\right] \tag{34}$$

where ζ_3 is simply given by

$$\zeta_3(\{k_i\}) = \zeta(k_1,k_2,k_3,k_4\to 0). \tag{35}$$

To express it in another way we still have

$$\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_3}\rangle = \delta_{\rm Dirac}\left(\sum\mathbf{k}_i\right)\,P_3(\{k_i\}) \tag{36}$$

$$P_3(\{k_i\}) = \nu_3(\{k_i\})\sum_i\prod_{j\neq i}\frac{H^2}{2k_j^3} \tag{37}$$

with a vertex value ν_3 to be taken in the appropriate limit, $k_4\to 0$. The shape dependence of the vertex, or the time dependance of this cumulant thus reproduces that of the fourth cumulant as shown on Figs. 1 (solid line) and 4.

IV. COMMENTS

We have obtained some closed forms for the 3- and 4-point correlation functions of a test scalar field with a self-interacting potential of order respectively 3 and 4 in a de Sitter background.

It is interesting to note, as it was already in the literature [8], that the superhorizon behaviour of the field can be obtained from a simplified Klein-Gordon equation for the field evolution on superhorizon scales

$$\ddot\chi + 3H\dot\chi = -\frac{\partial V}{\partial\chi}(\chi) \tag{38}$$

solved perurbatively at first order in λ. In this equation χ has to be understood as the filtered value of χ at a fixed scale that leaves the horizon at a given time t_0. The previous equation is then valid for $t > t_0$ only where the field can be described by a classical stochastic field. Moreover in writing this equation one also makes the assumption that

its r.h.s. can be computed from the filtered value of the field (which is not necessarily identical to what would have been obtained from a filtering of the source term). The equation (38) can be solved perturbatively in λ. At zeroth order χ is constant and, at first order, it reads

$$\chi^{(1)} = \chi_0^{(1)} - \frac{\partial V}{\partial \chi}\left(\chi^{(0)}\right)\frac{t-t_0}{3\,H}, \tag{39}$$

if $\chi_0^{(1)}$ is the leading order value of the field at horizon crossing. Note that $t-t_0$ can be rewritten as N_e/H where N_e is the number of e-folds since horizon crossing. It implies that if the number of e-folds is large enough the term $\chi_0^{(1)}$ should become negligible. As a consequence the leading order expression of the first nontrivial high order cumulant of the one-point PDF of χ takes either the form $3\,\nu_3\langle\chi^2\rangle^2$ or $4\,\nu_3\langle\chi^2\rangle^3$ for respectively a cubic or a quartic potential. The value of the coefficient ν_3 that enters these expressions is precisely the one obtained in Eq. (30) in the superhorizon limit, that is $\nu_3 = -\lambda\,N_e/(3\,H^2)$. And indeed if one had to compute such cumulants from either expression (29) or (37) the integration over the wave vectors would have led to the very same expressions in the superhorizon limit. That shows that the late time behavior of the cumulants we found comes in fact from a simpler dynamical evolution. It actually demonstrates that the computations in the classical approach sketched here can be put on a firm ground. It also allows investigations of more subtle effects, such as the finite volume effects where more than one filtering scales have to be taken into account, that cannot be properly addressed in a classical approach.

Regarding finite volume effect it is interesting to investigate how the results for the cubic and the quartic potentials could be related together. We refer here to our companion paper [13] where we give a comprehensive presentation of finite volume effects on observable quantities. Suffice is to say that even if one assumes χ has a self interacting quartic potential, a non-zero third order correlator might be observable. The reason is that one cannot have access to genuine ensemble averages but to constrained ensemble averages such as $\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_3}\rangle_{\overline{\chi}}$ which is the expectation value of $\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_3}$ for a given value of $\overline{\chi}$, average value of χ over the largest scale available in the survey in which the correlators are computed.

To leading order in $\overline{\chi}$ with respect to its variance, $\sigma_{\overline{\chi}}$, this constrained average value reads

$$\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_3}\rangle_{\overline{\chi}} = \langle\chi_{vk_1}\dots\chi_{\mathbf{k}_3}\,\overline{\chi}\rangle_c\,\frac{\overline{\chi}}{\sigma^2_{\overline{\chi}}}. \tag{40}$$

If, for $i = 1\dots3$, the wavelengths $1/k_i$ are much smaller than the survey size then the expression of $\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_3}\,\overline{\chi}\rangle_c\,\overline{\chi}/\sigma^2_{\overline{\chi}}$ is that of $\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_3}\rangle$ when the self-interacting field χ evolves in the potential $\lambda\overline{\chi}\,\chi^3/3!$, which is the cubic term in χ in the expansion of $\lambda\,(\chi+\overline{\chi})^4/4!$. This shows that the two cases described in this paper are consistent with one another and that the functional relation between the two had to be expected.

To conclude we have established in this paper a number of generic results that put the tree-order computation of higher order correlations in a de Sitter background on a secure ground. We leave for further studies the examination of more realistic cases in which the background expansion is more complex.

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[13] F. Bernardeau and J.-P. Uzan, [`arXiv:astro-ph/0311421`].
[14] Calculations in other background are difficult in general but we have checked that in case of a power law inflation, $a(t)\propto t^{\nu}$, we recover the same behaviors in the superhorizon limit if ν is large enough.

[15] It is to be noted that these calculations do not correspond to those of diffusion amplitudes of some interaction processes in a de Sitter space. When one tries to do these latter calculations with a path integral formulations, mathematical divergences are encountered as it has been stressed in [8, 12]. With that respect de Sitter space differs from Minkowski space-time. Our current point of view on this difficulty is that diffusion amplitudes cannot be properly defined in de Sitter space and that only field correlators as defined here correspond to actual observable quantities that can be safely computed.